

The Unmusical Ear: Georg Simon Ohm and the Mathematical Analysis of Sound

Melle Jan Kromhout, *Independent Scholar*

Abstract: This essay presents a detailed analysis of Georg Simon Ohm’s acoustical research between 1839 and 1844. Because of its importance in Hermann von Helmholtz’s subsequent study of sound and hearing, this work is rarely considered on its own terms. A thorough assessment of Ohm’s articles, however, can greatly enrich our understanding of later developments. Based on study of Ohm’s published writings, as well as a lengthy unpublished manuscript, the essay argues that his acoustical research foreshadows an important paradigmatic shift at a time of discursive instability prior to Helmholtz’s influential contributions. Using Ohm’s own dismissal of his supposedly “unmusical ears” as a conceptual frame, the essay describes this shift as a move away from understanding sound primarily in a musical context and toward an increasingly mathematical approach to sound and hearing. As such, Ohm’s work also anticipates a more general change in the role of the senses in nineteenth-century scientific research.

In the summer of 1843 Georg Simon Ohm—most famous for his law of electrical conduction of 1827—published one of the most influential articles in the history of modern acoustics. “On the Definition of a Tone with the Associated Theory of the Siren and Similar Sound Producing Devices” put forward the proposition that the application of Fourier’s theorem to the analysis of siren tones shows, first, that they contain a single sinusoidal frequency (what would later be known as a sine wave) corresponding to the fundamental pitch of the tone; and, second, that each tone is a compound mixture of smaller frequencies (overtones, harmonics, or partials) that can be represented by series of superpositioned sine and cosine values (a

Melle Jan Kromhout researches conceptual relations between music, sound, and media. Now an independent scholar, he completed a Ph.D. at the University of Amsterdam and worked as a postdoctoral fellow at the University of Cambridge. His monograph *The Logic of Filtering: How Noise Shapes the Sound of Recorded Music* is forthcoming with Oxford University Press. Admiraal de Ruijterweg 189-II, 1056GD Amsterdam, Netherlands; mellejankromhout@gmail.com.

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Fourier series).¹ Together, Ohm argued, these findings support the hypothesis that simple harmonic or sinusoidal motion constitutes the basis of all musical tones. At the time of their publication, however, few people recognized the importance of these findings. After a heated debate with the acoustician August Seebeck, Ohm did not pursue his acoustical research any further.

Only in 1856, two years after Ohm's death, did Hermann von Helmholtz take up the issue again in "On Combination Tones," which gave a detailed account of the dispute between Ohm and Seebeck and concluded that Ohm's application of Fourier analysis and his assumptions about the sinusoidal shape of elemental tones had been correct. These claims, substantiated by extensive empirical work, were furthered in Helmholtz's *On the Sensations of Tone as a Physiological Basis for the Theory of Music*, published in 1863.² Its use in the "resonance theory" of hearing that was introduced in this book secured the legacy of Ohm's contribution.³ Because of the enormous influence of this work on subsequent developments, however, assessments of Ohm's acoustical articles are mostly motivated by their use in Helmholtz's analysis of sound and hearing, whereas the perspective on Ohm's work as such remains somewhat obscured.⁴ With this essay, I want to show that a closer assessment of Ohm's acoustic work, considered on its own terms, can significantly enrich our understanding, first, of Helmholtz's contributions to modern acoustics and psychoacoustics and, second, of a broader change in European scientific culture over the course of the nineteenth century, as exemplified by the changing role of the ear as a tool for and object of research.

Introduced by Jean Baptiste Joseph Fourier in his *Analytical Theory of Heat* of 1822, Fourier's theorem mathematically transforms a periodic function representing the development of some signal over time (heat propagation, the vibrations of a string, tidal movements, the circulations of heavenly bodies) in terms of converging series of sine and cosine values that

¹ Georg Simon Ohm, "On the Definition of a Tone with the Associated Theory of the Siren and Similar Sound Producing Devices," in *Acoustics: Historical and Philosophical Development*, ed. R. Bruce Lindsay (Stroudsburg: Dowden, Hutchinson & Ross, 1972), pp. 242–247, on p. 244. This is a partial translation of Ohm, "Über die Definition des Tones, nebst daran geknüpfter Theorie der Sirene und ähnlicher tonbildender Vorrichtungen," *Annalen der Physik und Chemie*, 1843, 2nd Ser., 59:513–565.

² Hermann von Helmholtz, "Ueber Combinationstöne," *Ann. Phys. Chem.*, 1856, 2nd Ser., 99:497–540; and Helmholtz, *On the Sensations of Tone as a Physiological Basis for the Theory of Music*, trans. Alexander J. Ellis (1875; Cambridge: Cambridge Univ. Press, 2012), <https://doi.org/10.1017/CBO9780511701801>.

³ With his "resonance" or "place" theory of hearing, Helmholtz argued that the tiny fibers on the basilar membrane in the inner ear resonate with individual frequencies, thus operating as a tuned resonator. He thereby suggested that the inner ear essentially works as a Fourier analyzer, decomposing compound sounds into simple elements. See R. Plomp, *Experiments on Tone Perception* (Soesterberg: Institute for Perception RVO-TNO, 1966), p. 103. See also Eric J. Heller, *Why You Hear What You Hear: An Experiential Approach to Sound, Music, and Psychoacoustics* (Princeton, N.J.: Princeton Univ. Press, 2012), p. 443; and Alain de Cheveigné, "Pitch Perception Models," in *Pitch: Neural Coding and Perception*, ed. Christopher J. Plack *et al.* (New York: Springer, 2005), pp. 169–233, esp. pp. 178–183.

⁴ Ohm's acoustic work was acknowledged only after Helmholtz's publications; it was not mentioned, for instance, in a memorial speech in 1855: Johann von Lamont, *Denkrede auf die Akademiker Dr. Thaddäus Siber und Dr. Georg Simon Ohm* (Munich: Verlag der Königlichen Akademie, 1855). Dayton Miller's definition of "Ohm's acoustic law" (1937), as quoted by Eric Heller, states that "all musical tones are periodic functions; that the ear perceives pendular [sinusoidal] vibrations alone, as simple tones; that all varieties of tone quality or tone color are due to particular combinations of a larger or smaller number of simple tones of commensurable frequencies; and that a complex musical tone or a composite mass of musical tones is capable of being analyzed into a sum of simple tones." In fact, only the first and the last parts of this definition correspond more or less with Ohm's findings, whereas the claim about tone color is by Helmholtz. See Heller, *Why You Hear What You Hear*, p. 444. More recent discussions of Ohm's acoustic work—for instance, by Benjamin Steege or Julia Kursell—focus on his dispute with Seebeck, which is discussed primarily in light of their larger argument about Helmholtz. See Benjamin Steege, *Helmholtz and the Modern Listener* (Cambridge: Cambridge Univ. Press, 2012), pp. 46–54; and Julia Kursell, *Epistemologie des Hörens: Helmholtz' physiologische Grundlegung der Musiktheorie* (Berlin: Fink, 2018), pp. 151–153. The same can be said of R. Steven Turner's excellent article on the topic: R. Steven Turner, "The Ohm–Seebeck Dispute, Hermann Von Helmholtz, and the Origins of Physiological Acoustics," *British Journal for the History of Science*, 1977, 10:1–24 (hereafter cited as Turner, "Ohm–Seebeck Dispute").

represent partial states of its distribution in space.⁵ Although its genesis reaches back, as we shall see, to the debate on the representation of a vibrating string in the eighteenth century, these acoustical origins were almost entirely absent in Fourier's treatise and its initial reception in mathematical and physical circles.⁶ Two decades later, Ohm brought Fourier analysis back to acoustics. He showed that, in the case of periodic sound waves, the sine and cosine values correspond to the amplitude, phase, and frequency of every partial or overtone. Significantly, because the most straightforward type of Fourier analysis, as applied by Ohm and Helmholtz, presupposes absolute periodicity, it excludes any sense of temporal development: it represents sound spectra as series of entirely static, infinite oscillations.⁷ As Tara Rodgers points out, Ohm's article thereby marks a key moment in the history of acoustics, at which "the sinusoidal form was resolved to be the fundamental material and most common representational building block of all sounds."⁸ As an integral part of Helmholtz's resonance theory, Ohm's application of Fourier analysis and the resulting idea of the sine wave as "elemental tone" became cornerstones of modern acoustics and psychoacoustics.

A study of this particular moment, placing Ohm's acoustic articles in the context of his larger body of work and the intellectual climate in which they were conceived, therefore provides crucial insights into the way in which the science of sound was transformed between the eighteenth-century mechanics of the vibrating string and Helmholtz's nineteenth-century psychophysical work on sound and hearing. After briefly outlining Ohm's background, the essay first discusses the troublesome reception of his research on electricity in the 1820s. Heavily influenced by Fourier's treatise, this work exemplifies Ohm's innovative style of mathematical physics and foreshadows a broader scientific shift from physical research that was based primarily on empirical observation to research that relied increasingly on mathematical models. This shift also characterizes Ohm's work in acoustics between 1839 and 1843. On the basis of publications by Ohm and his contemporaries, as well as the study of a lengthy unfinished and unpublished manuscript currently kept in the archive of the Deutsches Museum in Munich, I argue that, rather than being prompted by any specific interest in acoustics, let alone in music, the primary appeal of these issues for Ohm was mathematical.

Like Fourier before him, Ohm worked in an unstable and rapidly changing field in between eighteenth-century mathematics and mechanics and nineteenth-century mathematical physics, the disciplinary borders and epistemological premises of which had yet to be clearly defined. The study of just such transitional moments, when the status and meaning of new methods and concepts are being developed and renegotiated, can shed light on the conditions and conceptual assumptions that produced new scientific discourses. Because of the durable legacy of Helmholtz's work, many of the preconceptions and assumptions that shaped Ohm's analysis continue to carry weight to the present day.⁹ To frame these discursive resonances of

⁵ Jean Baptiste Joseph Fourier, *The Analytical Theory of Heat*, trans. Alexander Freeman (1878; Cambridge: Cambridge Univ. Press, 2009).

⁶ Olivier Darrigol, "The Acoustic Origins of Harmonic Analysis," *Archive for History of Exact Sciences*, 2007, 61:343–424, esp. p. 345. On the immediate reception of Fourier's treatise see Elizabeth Garber, "Reading Mathematics, Constructing Physics: Fourier and His Readers, 1822–1850," in *No Truth Except in the Details: Essays in Honor of Martin J. Klein*, ed. A. J. Kox and Daniel M. Siegel (Dordrecht: Kluwer Academic, 1995), pp. 31–54.

⁷ In the case of nonperiodic signals, the Fourier transform treats the entire signal, or some part of it, as one infinitely long cycle and derives the frequency spectrum of this symbolically infinite cycle not by summing but by integrating the sine and cosine values. In nineteenth-century sound analysis, however, only the straightforward form of Fourier analysis, applied to strictly periodic sounds, was used. See Daniel Muzzolini, "Genealogie der Klangfarbe" (Ph.D. diss., Univ. Zurich, 2004), p. 348.

⁸ Tara Rodgers, "Synthesizing Sound: Metaphor in Audio-Technical Discourse and Synthesis History" (Ph.D. diss., McGill Univ., 2010), p. 124.

⁹ According to Alain de Cheveigné, "the place theory of Helmholtz is still used in at least four areas: (1) to explain pitch of pure tones (for which objections are weaker), (2) to explain the extraction of frequencies of partials . . . , (3) to explain spectral pitch . . . , and (4) in textbook accounts": Cheveigné, "Pitch Perception Models" (cit. n. 3), p. 182.

Ohm's work conceptually in modern acoustics, I draw on the author's dismissal (in the final entry of his dispute with Seebeck, from 1844) of his supposedly "unmusical" ears.¹⁰ Through this notion of the "unmusical ear," I show how Ohm's acoustical work played an important part in the negotiation between the increasingly mathematical field of acoustics and a long-standing tradition that placed most thinking about sound in the context of music.

Despite established notions of musicality and the need for a "good" musical ear, Ohm was not at all concerned with or interested in music; nor, unlike the very musically minded Helmholtz, was he much concerned with psychological questions of hearing or with the physiology of the ear. Ohm's aim was to establish a physical definition of a tone on the basis of mathematical analysis: the mathematics came first, the sound only second. We might therefore read Ohm's self-confessed "unmusical ear" as a sign of a paradigmatic shift in the role of sensation in scientific research: it pivots away from a culture that understood sound primarily in the context of music and (via the mathematical study of sound *as such* with Ohm) toward a more nuanced understanding of the role of the listening subject itself with Helmholtz and beyond. Ultimately, Ohm's "unmusical ear" marks the beginning of an age in which human hearing would be increasingly undercut, augmented, and shaped by technologies that, by virtue of mathematical physics, are based on nonhuman sound analysis—or, indeed, on unmusical ears.

FROM GALVANIC CIRCUIT TO COMBINATION TONES

Born in 1789 in Erlangen, Bavaria, Georg Simon Ohm was raised by a locksmith with a keen interest in mathematics, science, and philosophy who made sure his two sons received a solid education through home schooling and at the local *Gymnasium*. The fruits of this upbringing are evident from young Georg's correspondence when working as a private teacher in Switzerland in 1806.¹¹ While Georg exchanged mathematical problems with his brother Martin (who would become a respected professor of mathematics in Berlin), Ohm's father encouraged his older son to read the transcendental idealism of Immanuel Kant and Johann Gottlieb Fichte as well. Fichte's work in particular influenced Ohm's early views on science and education, as is evident from the introduction to his first publication, a textbook on geometry published while he worked as a teacher of mathematics and physics in Cologne in 1817. Contrary to the dominant practice of learning mathematics by doing exercises, Ohm's method aimed to stimulate a "transition from intuiting to thinking" in students. He did not consider learning geometry an educational goal in itself but, rather, the very foundation of any scientific education—a "means of forming the power of thought."¹² The book thereby shows how his understanding of the role of mathematics as a driving force in the study of natural phenomena already distinguished him from many of his contemporaries.

This difference became more pronounced in the experimental research he took up next, detailed accounts of which first appeared as a series of articles in 1825 and 1826.¹³ Although

¹⁰ Georg Simon Ohm, "Noch ein Paar Worte über die Definition des Tones," *Ann. Phys. Chem.*, 1844, 2nd Ser., 62:1–18, on p. 7.

¹¹ Many of these letters are reproduced in Georg Simon Ohm, *Nachgelassene Schriften und Dokumente aus seinem Leben*, ed. Walter Füchtbauer and Bernd Nürnberg (Erlangen: Palm und Enke, 2002).

¹² Georg Simon Ohm, *Grundlinien zu einer zweckmäßigen Behandlung der Geometrie als höheren Bildungsmittel an vorbereitende Lehranstalten* (Erlangen: Palm und Enke, 1817), pp. v, ix (translations of German sources are my own if not otherwise attributed).

¹³ Georg Simon Ohm, "Vorläufige Anzeige des Gesetzes, nach welche Metalle die Contact-Electricität-leitern," *Journal für Chemie und Physik*, 1825, 44:79–88; Ohm, "Bestimmung des Gesetzes, nach welchem Metalle die Contactelectricität leiten, nebst einem Entwurfe zu einer Theorie des Voltaischen Apparates und des Schweigger'schen Multiplicators," *ibid.*, 1826, 16:137–166; and Ohm, "Versuch einer Theorie der durch galvanische Kräfte hervorgebrachten elektroskopischen Erscheinungen," *Ann. Phys. Chem.*, 1825, 2nd Ser., 4:79–88.

the second of these already contained his famous law of electrical conduction, the articles did not attract much attention. Ohm therefore decided to outline and further develop his discoveries in a book; and since the experimental results had already been published in the articles, *The Galvanic Circuit Investigated Mathematically* (1827) focused exclusively on their mathematical development. By showcasing a shift away from qualitative empirical results supported by mathematical theory toward a more central role for mathematical analysis, the book is a testament to the novelty of Ohm's approach. In this work, explicitly modeled on that of the late eighteenth- and early nineteenth-century French mathematical physicists that he studied extensively in his youth (Laplace, Poisson, Fresnel, and, above all, Fourier), Ohm did not simply use mathematics to interpret his experimental physical results. Instead, as he writes in the introduction, he sought "to secure incontrovertibly to mathematics the possession of a new field of physics, from which it had hitherto remained almost totally excluded."¹⁴ For Ohm, then, mathematical analysis was not just a tool but a goal in itself.

This focus reflects Ohm's position as an early representative of a major transition in German scientific culture between the 1820s and the 1850s, which Kenneth Caneva characterizes as the development from a generation of "concretizing scientists" in the first decades of the century toward a generation of "abstracting scientists" from the 1830s onward. For the older group, empirical research and qualitative results based on close observation were primary, while mathematics was used to highlight and further explicate experimental results. Scientists were often self-taught, and their work did not yet adhere to the clear disciplinary distinctions of the new university system that developed in Germany in the first decades of the century. The younger generation, on the other hand, were academically trained specialists who no longer used advanced mathematics only to explicate qualitative empirical results but also to develop new models and deduce hypotheses that were subsequently verified with specialized experimental setups for detailed quantitative measurement.¹⁵

Although Ohm was a contemporary of the "concretizing" generation, his work displays many facets of the "abstracting" approach.¹⁶ This was largely due to his deep knowledge of mathematical-physical research from France and especially Fourier's *Analytical Theory of Heat*, whose approach Ohm adopted in *The Galvanic Circuit*.¹⁷ Because he suspected a physical analogy between heat propagation and electrical conduction, Fourier's treatise inspired him to highlight the mathematical analysis instead of further explicating his experimental results.¹⁸ Precisely this mathematical focus and its separation from the experimental data,

¹⁴ Georg Simon Ohm, *The Galvanic Circuit Investigated Mathematically*, trans. William Francis (New York: Van Nostrand, 1891), p. 17. See also Elizabeth Garber, *The Language of Physics: The Calculus and the Development of Theoretical Physics in Europe, 1750–1914* (New York: Springer, 1999), pp. 159–177. As he writes in a letter in 1826, Ohm's explicit goal was indeed "to gain solid ground in a field where in recent times . . . Frenchmen seem to have become the sole rulers": Jos. Schnippenkötter, "Ohm in Köln: Beiträge zur Geschichte der Mathematik und Physik zu Beginn des 19. Jahrhunderts," in *Georg Simon Ohm als Lehrer und Forscher in Köln 1817 bis 1826*, ed. Hans Vogts et al. (Cologne: Bachem, 1939), pp. 63–172, on p. 153 (quoting Ohm).

¹⁵ Kenneth L. Caneva, "From Galvanism to Electrodynamics: The Transformation of German Physics and Its Social Context," *Historical Studies in the Physical Sciences*, 1978, 9:63–159, on p. 66; and Christa Jungnickel and Russell McCormach, *Intellectual Mastery of Nature: Theoretical Physics from Ohm to Einstein*, Vol. 1 (Chicago: Univ. Chicago Press, 1986), pp. 3–33.

¹⁶ Caneva, "From Galvanism to Electrodynamics," p. 66; and Garber, "Reading Mathematics, Constructing Physics" (cit. n. 6), p. 39.

¹⁷ As Garber writes: "Ohm was not merely influenced by Fourier but annexed pages of Fourier to solve the problem of reducing the galvanic circuit to mathematics. He then solved the mathematical equation following the methods of Fourier and other French mathematicians." See Garber, *Language of Physics* (cit. n. 14), pp. 177–178. See also Bernard Pourpux, "De la reconstitution de la physique allemande du XIXe siècle: Les exemples de Georg Simon Ohm et Hermann Helmholtz," *Revue d'Histoire des Sciences*, 2007, 60:185–202.

¹⁸ Regarding this analogy between heat and electricity, Brian Gee writes that "in some respects he was misled, as pointed out by James Clerk Maxwell, yet he finished up with the correct final result for steady conduction": Brian Gee, "Georg Simon Ohm, 1789–1854," *Physics Education*, 1969, 4(2):106–113, on p. 111.

however, caused the book to be as poorly received as the earlier articles, with many critics arguing that Ohm's abstract mathematical account did not suffice to explain the natural phenomena it was said to represent.¹⁹

Also contributing to the poor reception of Ohm's work was the fact that the validity and general applicability of Fourier's theorem were still heavily debated. The unresolved question of whether the theorem offered solutions only in special cases or stood as a general solution already caused serious objections when Fourier presented his theory to the Académie Française, first in 1807 and again in 1811.²⁰ Even after its publication in 1822, it remained to be seen whether the theorem was as general as Fourier suggested. Only in 1829, two years after the publication of *The Galvanic Circuit*, did the mathematician Johann Peter Gustav Lejeune Dirichlet prove the general mathematical validity of Fourier's theorem, thereby settling a century-long dispute regarding the general applicability of trigonometric series to discontinuous and nonperiodic functions.²¹ Because of its complicated mathematics, however, Dirichlet's proof initially met with the same slow reception in nonmathematical circles as had *The Galvanic Circuit* two years earlier.

In the meantime, Ohm had become increasingly disillusioned by the lack of recognition. By the early 1830s frustration and his failure to secure a university professorship led him to abandon research altogether. He relocated to Nuremberg to become a professor at the polytechnic school—initially teaching physics, subsequently also mathematics, and ultimately becoming head of the institute in 1838. Although he did not publish anything more until the end of the decade, his notebooks show that he kept up with debates in mathematics and physics throughout the 1830s, while also returning regularly to Fourier's treatise. Alongside abstracts of the work of Dirichlet, Augustin-Louis Cauchy, and others, he made at least four summaries of Fourier's treatise between 1825 and 1835.²² Given this continued engagement with Fourier's work, it comes as no surprise that Ohm's return to scholarly work in the late 1830s included a new application of Fourier's theorem: the harmonic analysis of siren tones.

After his move to Nuremberg, Ohm's law on electrical conduction began to gain wider recognition in Germany and abroad. This ultimately encouraged him to resume publishing.²³

¹⁹ Ernst Georg Deuerlein, *Georg Simon Ohm, 1789–1854: Leben und Wirken des grossen Physikers* (Erlangen: Palm und Enke, 1954), p. 14. Morton L. Schagrin attributes this poor reception not so much to the mathematical complexity itself, but to its conceptual shift in the study of electricity: Morton L. Schagrin, "Resistance to Ohm's Law," *American Journal of Physics*, 1963, 31:536–547.

²⁰ The 1811 committee, consisting of Lagrange, Laplace, Malus, Haüy, and Legendre, remarked that Fourier's solutions left "something to be desired on the score of generality and even rigour": committee report cited in John Herivel, *Joseph Fourier: The Man and the Physicist* (Oxford: Clarendon, 1975), p. 103.

²¹ M. Norton Wise, "What's in a Line?" in *Cultures and Politics of Research from the Early Modern Period to the Age of Extremes*, Vol. 1: *Science as Cultural Practice*, ed. Moritz Epple and Claus Zittel (Berlin: Akademie, 2010), pp. 61–102, esp. p. 80. Notably, Dirichlet was a student of Ohm in Cologne and later studied with Fourier in Paris.

²² The first page of the first notebook (Archiv des Deutschen Museums, Munich, NL267/018), which the archive dates between 1825 and 1830, says "13 February 1825." Its summary of Fourier is approximately one hundred pages long. The first page of the second summary (in NL267/022) says "21 October 1829." Like the third summary (also in NL267/022), it is some sixty to seventy pages long. Notably, this volume also contains notes on Dirichlet, dated 8 Nov. 1829, and several other works on differential calculus, including works by Cauchy from 1823 and Lacroix from 1819. It concludes with extensive notes on eighteenth-century calculus, mentioning Jacob Bernoulli, d'Alembert, Euler, and Lagrange. The fourth and final Fourier summary (in NL267/023) is only seven pages long and mostly consists of direct citations in French. Most of the other material in this notebook, dated between 1830 and 1835, was published prior to 1830, and it seems safe to assume that the abstracts were written between 1830 and 1832. They all deal with trigonometric series, including Dirichlet's "Note sur intégrales définies" (1829) and a memoir by Cauchy from 1823.

²³ Kenneth Caneva, "Ohm, Georg Simon," in *Encyclopedia.com*, 2016, <http://www.encyclopedia.com/people/science-and-technology/physics-biographies/georg-simon-ohm> (accessed 8 Feb. 2017).

The short article “Remarks on Combination Tones and Beats,” published in the summer of 1839, presented an analysis of the lower “third tone” that can be heard when two tones are played simultaneously with sufficient intensity. Ever since its discovery in the mid-eighteenth century, most authors had believed this phenomenon to be related to the “beats” or rapid “beating” sound that occurs when two tones of slightly different frequency sound together.²⁴ Early in the nineteenth century Thomas Young, for instance, advocated this “beat-theory” of combination tones. His dispute on the matter with the Scottish botanist John Gough was summarized in German in 1805 by Gerhard Vieth, who also coined the term “combination tone.”²⁵

This exchange between Young and Gough brought up a number of issues that would remain important throughout nineteenth-century acoustics. Whereas Young assumed combination tones to be objective phenomena with a physical origin, Gough thought they were the product of our subjective hearing experience because their vibrations could not be empirically observed. He thereby suggested, for the first time, that our hearing mechanism itself might influence what we hear.²⁶ As Vieth recognized, given this emphasis on the role of the ear itself, the different explanations of combination tones put forward by Young and Gough pointed to different understandings of what constitutes a “tone” in the first place. Although both authors agreed that most tones consist of multiple “smaller” tones, they entertained different ideas regarding the relation between the whole and its parts. Young considered compound tones to be simple entities consisting of a coalescence of “smaller” tones, but he remained unclear about the way in which, as Carlton Maley paraphrases his uncertainty, “the ear could deduce the ‘complicated idea of heterogeneous vibrations’ from this simplicity.” Gough, on the other hand, did not accept this idea of “simple” compounds as aggregates of smaller tones. He posited that compound tones are complex “mixtures” that the ear cannot separate into individual components.²⁷

These different definitions of compound tones and the question of the role of sensory perception foreshadowed the dispute between Ohm and Seebeck some four decades later but originated in eighteenth-century discussions on the mathematical representation of a vibrating string. It is precisely this connection to the far-reaching mathematical analysis of vibrating strings that further explains Ohm’s interest in the topic. Whereas eighteenth-century analyses of combination tones focused primarily on musical questions regarding harmony, intonation, and instrument tuning, this focus slowly gave way to more mathematical interpretations, especially after Joseph-Louis Lagrange linked the issue to Joseph Sauveur’s theory of beats.²⁸ This mathematical approach really caught on around 1800, when Young explained both combination tones and beats in the context of his own theory of wave interference. Given Ohm’s well-documented knowledge of eighteenth-century French mathematical physics and the fact that,

²⁴ Georg Simon Ohm, “Bemerkungen über Combinationstöne und Stösse,” *Ann. Phys. Chem.*, 1839, 2nd Ser., 47:463–466. The German organist Andreas Sorge first reported on combination tones in 1744, the French scientist Jean Baptiste Romieu in 1751, and, most extensively, the Italian violinist and music theorist Giuseppe Tartini in 1754, although he claimed to have discovered the phenomenon as early as 1714. See V. Carlton Maley, Jr., *The Theory of Beats and Combination Tones, 1700–1863* (New York: Garland, 1990), pp. 36–41. Maley’s book is the most complete overview of the development of theories of combination tones and beats up to Helmholtz.

²⁵ Gerhard Vieth, “Ueber Combinationstöne, in Beziehung auf einige Streitschriften über die zweier Englischer Physiker, Th. Young und Jo. Gough,” *Ann. Phys. Chem.*, 1805, 2nd Ser., 11:265–314.

²⁶ Kursell, *Epistemologie des Hörens* (cit. n. 4), p. 40.

²⁷ Vieth, “Ueber Combinationstöne” (cit. n. 25), pp. 313–314; Maley, *Theory of Beats and Combination Tones* (cit. n. 24), p. 74; and John Gough, “The Theory of Compound Sounds,” *Journal of Natural Philosophy, Chemistry, and the Arts*, 1803, 4:152–159, esp. p. 158.

²⁸ Joseph-Louis Lagrange, “Recherches sur la nature et la propagation du son,” in *Oeuvres de Lagrange*, Vol. 1 (Paris: Gauthier-Villars, 1867), pp. 39–148.

on the other hand, there are no records, biographical or archival, of any intensive engagement with acoustics prior to 1839 or after 1844—and no proof whatsoever of a solid musical education or even interest in music at any point in his life—it seems likely that this mathematical perspective sparked his interest in combination tones as well.²⁹

In 1829, after Vieth's summary of the dispute between Young and Gough, Wilhelm Weber, who had published a hefty study of the mechanics of waves with his brother Ernst in 1825, wrote a short piece on combination tones.³⁰ The article, which still supported the beat theory, showcased a more mathematical approach, including a table of different combination tones expressed in frequency numbers instead of musical pitches. Weber's article, in turn, prompted the Swedish scientist Gustav Hällström to revise and translate his dissertation from 1819. Printed in the *Annalen* in 1832, Hällström's text presented the most extensive theory of combination tones since Lagrange and Young. Significantly, even as he continued to promote the beat theory, Hällström also speculated about the existence of so-called higher-order combination tones. Besides the tone produced by the primary interval, he suggested, the interference between the resulting combination tone and the primary tone would produce yet another, "second-order," combination tone, which in turn triggers a "third-order" tone, and so on, *ad infinitum*. Hällström's tables of these higher-order combination tones listed musical pitches and intervals but also detailed frequencies.³¹

Although published seven years later, Ohm's "Remarks on Combination Tones and Beats" was a direct response to Hällström. If not for the importance of his subsequent work on acoustics, it would probably have been forgotten. In light of these future developments, however, it introduced two important points. First, Ohm suggested that higher-order combination tones are not caused by the interference of primary and resultant tones but, rather, by the interference between "the harmonic tones accompanying the original tones." In other words, he argued that higher-order combination tones are produced by coalescing overtones. If this was the case, Ohm speculated, higher-order combination tones would not, as Hällström suggested, appear in every case but would only accompany tones with *harmonic* overtones, such as those produced by vibrating strings and columns of air. When the overtones are not harmonics of the fundamental, as is the case with oscillating rods like tuning forks, no higher-order combination

²⁹ Ohm's notebooks contain one abstract on sound (in Archiv des Deutschen Museums, NL267/018): Poisson's "Mémoire sur la théorie du son" (1808). Given the other abstracts in this volume, and the fact that it is directly followed by the first of the four Fourier summaries (see note 22, above), however, Ohm's primary interest seems to have been trigonometric series. Still, he was well aware of the mechanics of acoustics as they had been established by the 1830s and 1840s, as is evident from the section on acoustics in his collected lectures: Georg Simon Ohm, *Grundzuege der Physik als Compendium zu seinen Vorlesungen* (Nuremberg: Joh. Leonh. Schrag, 1854), pp. 164–178. Regarding biographical material, the only extensive—though rather hagiographic—biography is Heinrich von Füchtbauer, *Georg Simon Ohm: Ein Forscher wächst aus seiner Väter Art* (Berlin: VDI, 1939). Additionally, Füchtbauer and Nürnberger offer extensive editorial notes in Ohm, *Nachgelassene Schriften und Dokumente aus seinem Leben* (cit. n. 11). Further biographical sources include Ohm, *Aus Georg Ohms handschriftlichem Nachlass: Briefe, Urkunden und Dokumente*, ed. Ludwig Hartmann (Munich: Bayerland, 1927); Friedrich Mann, *Georg Simon Ohm* (1890), trans. Irmeli Kuehnel (Seattle: Pentode, 2007) (which contains the memories of a former student of Ohm); Vogts *et al.*, eds., *Georg Simon Ohm als Lehrer und Forscher in Köln* (cit. n. 14); Deuerlein, *Georg Simon Ohm, 1789–1854* (cit. n. 19); E. Mollwo, *Georg Simon Ohm: Leben und Wirken* (Erlangen: Georg-Simon-Ohm-Verein, 1980); and Peter May, *Georg Simon Ohm: Leben und Wirkung* (Erlangen: Mayer, 1989).

³⁰ Wilhelm Weber, "Ueber die Tartinischen Töne," *Ann. Phys. Chem.*, 1829, 2nd Ser., 15:216–222. Weber's article was inspired by a report by the Frenchman Baron Blein, published two years earlier.

³¹ Gustav Gabriel Hällström, "Von den Combinationstönen," *Ann. Phys. Chem.*, 1832, 2nd Ser., 24:438–466. See also Myles W. Jackson, *Harmonious Triads: Physicists, Musicians, and Instrument Makers in Nineteenth-Century Germany* (Cambridge, Mass.: MIT Press, 2006), p. 175.

tones would appear. Because there were no techniques to determine overtone series reliably, however, Ohm could not verify this suggestion empirically.³²

Ohm's second contribution was a tentative rejection of the relation between beats and combination tones. In contrast to the confidently argued first half of the article, however, this rejection was mostly unsubstantiated. Ohm admits that, because of the difficulties he encountered, he does not yet feel confident enough to share his calculations. In retrospect, both this rejection of the beat theory and the suggested role of overtones anticipated Helmholtz's work on combination tones, which would confirm that harmonic overtones are responsible for higher-order combination tones and also disprove the beat theory once and for all.³³ Most important for present purposes, Ohm's approach and presentation in this first acoustic article support my earlier claim that it was the mathematical challenge rather than a specific interest in acoustics that drew him to the problem of combination tones. This is further substantiated by an unfinished and unpublished manuscript in the archive in Munich, labeled "Elaborations on Combination Tones and Sound Waves."³⁴

Consisting of approximately two hundred handwritten pages, this document offers further clues as to why Ohm chose to work on acoustics and how the first article on combination tones led to the subsequent work on the definition of a tone. The decision to withhold his calculations in 1839 already suggested the expectation of some future publication, and this still seems to have been the plan when a footnote in "On the Definition of a Tone" in 1843 mentioned some upcoming "work on combination tones."³⁵ In all likelihood, the unpublished manuscript is what remains of this planned larger treatise. Some of the material might date back to the first article, although none of it seems to correspond directly to that publication. Since the manuscript also contains references to publications *after* 1839, it seems clear that Ohm continued working on this material long after publishing the first piece. Combination tones are the dominant focus throughout, but, significantly, about twenty pages are very similar and at times even identical to the paragraphs in "On the Definition of a Tone" that explain how Fourier analysis shows that simple harmonic motion is the basis of all tones.³⁶ Moreover, the document explicitly mentions the publication of "On the Definition of a Tone" at three separate instances (once in the main text and twice in marginalia).³⁷ The manuscript—developed alongside the published articles, between 1839 and at least 1843—thereby constitutes a "missing link" that

³² Ohm, "Bemerkungen über Combinationstöne und Stösse" (cit. n. 24), p. 465; and Maley, *Theory of Beats and Combination Tones* (cit. n. 24), p. 107.

³³ Maley, *Theory of Beats and Combination Tones*, p. 107; and Helmholtz, "Ueber Combinationstöne" (cit. n. 2), pp. 501, 529–539. To corroborate his theory of combination tones, Helmholtz conducted the very experiment to determine overtone series Ohm suggested in 1839.

³⁴ "Ausarbeitungen zu Kombinationstönen und Schallwellen," Archiv des Deutschen Museums, NL267/040. The manuscript consists of five unbound packets of (mostly) hand-numbered pages. These contain several consecutive but incomplete draft versions of the same material. I will refer to the order in which the packets are currently kept in the archive, although, based on their content, I would say that the correct chronological order is 3, 1, 2, 4, 5. The pages of packets 1–4 are numbered "R.1," "R.2," etc., whereas packet 5 uses "A.1."

³⁵ Ohm, "Über die Definition des Tones" (cit. n. 1), p. 558. Seebeck refers to the unpublished article in the final entry of his dispute with Ohm: August Seebeck, "Ueber die Definition des Tones," *Ann. Phys. Chem.*, 1844, 2nd Ser., 63:353–368, on p. 367. Turner indeed argues that the reason Ohm did not publish his calculations was that "he planned to publish a longer paper on the topic that never materialized," and he speculates that Seebeck "may have learned of Ohm's intention through private correspondence," although I would say that he might simply have been referring to Ohm's footnote. See Turner, "Ohm–Seebeck Dispute," p. 23.

³⁶ This section is in the first packet, paragraphs 8–12, pp. R.16–R.35. The published version is in Ohm, "Über die Definition des Tones," pp. 518–522.

³⁷ These references are found in the first packet, paragraph 8, p. R.19, and paragraph 12, p. R.33; and in the third packet, paragraph 12 (no page number).

allows us to track the intellectual development from the analysis of combination tones to the research on the definition of a tone.³⁸

As mentioned above, one of the most significant aspects of the first article was Ohm's hypothesis regarding the role of higher harmonics in the creation of combination tones. Although he neither published his calculations in 1839 nor performed his suggested experiment comparing sounds with harmonic or nonharmonic overtones, Ohm must have had some ideas about how his claim could be substantiated. Overtones had been observed and described at least since Marin Mersenne in the seventeenth century and Joseph Sauveur in the eighteenth, but their origin and nature remained heavily debated until well into the nineteenth century.³⁹ In the mid-eighteenth century, Daniel Bernoulli first proposed the idea that the vibrations of a string can be represented by series of superpositioned sines and cosines. At the time, however, his colleagues Jean le Rond d'Alembert and Leonhard Euler considered this analysis to be too general. Only with Fourier and Dirichlet was Bernoulli's approach proven correct—and indeed Ohm's problem in the first article (How might one determine the overtones of a given tone?) can be addressed via Fourier analysis.⁴⁰

The heavy intertwining in the unpublished manuscript of the issue of combination tones and the question of the definition of a tone shows that Ohm must have assumed a connection between the two issues early on; and he was not the first to do so. In two articles from 1834 and 1839, August Röber already remarked on the interdependence of the problem of combination tones and the definition of a tone more generally.⁴¹ We know that Ohm read the first of these, as Johann Christian Poggendorff's editorial commentary accompanying Röber's article is mentioned in his notes.⁴² Röber's second article appeared only a month after Ohm's initial publication. Besides a lengthy summary of the theory of combination tones (primarily championing the beat theory), it advances the idea that the "regular repetition" of "any kind of impulse"—regardless of its waveform—can generate a clearly pitched tone, as apparently shown by Seebeck's siren experiments of 1837.⁴³ As we shall see, disproving precisely this claim by Röber and Seebeck was the prime objective of "On the Definition of a Tone."

Hence, all the issues that inspired "On the Definition of a Tone" originate in the discussion on combination tones.⁴⁴ Moreover, the unpublished notes suggest that the second article was

³⁸ The notes can also be dated through a draft letter written on the first page of the third packet (also transcribed in Ohm, *Aus Georg Ohms handschriftlichem Nachlass* [cit. n. 29], pp. 194–195). The letter concerns the widow and children of one of Ohm's colleagues, Professor Kuppler, who died on 25 Sept. 1842. Above the draft letter, it reads "Combination tones of these simple tone ratios that have been subjected to observation by Hällström." Although this page lacks Ohm's own page and paragraph numbering and might therefore be misplaced, the combination of the title, the letter, and the date of Kuppler's death attest that Ohm was working on combination tones in September 1842. Furthermore, although the notes do not refer to any literature published after 1842, the references to "Über die Definition des Tones" indicate that Ohm was still working on the manuscript as late as early 1843.

³⁹ On harmonic overtones see Penelope Gouk, "The Role of Harmonics in the Scientific Revolution," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge Univ. Press, 2002), pp. 223–245; H. Floris Cohen, *Quantifying Music: The Science of Music at the First Stage of the Scientific Revolution, 1580–1650* (Dordrecht: Reidel, 1984); and John T. Cannon and Sigalia Dostrovsky, *The Evolution of Dynamics: Vibration Theory from 1687 to 1742* (New York: Springer, 1984).

⁴⁰ Darrigol, "Acoustic Origins of Harmonic Analysis" (cit. n. 6), p. 401.

⁴¹ August Röber, "Untersuchungen des Hrn. Scheibler in Crefeld über die sogenannten Schläge, Schwebungen oder Stösse," *Ann. Phys. Chem.*, 2nd Ser., 1834, 32:333–362; and Röber, "Combinationstöne und Stösse," *Repertorium der Physik*, 1839, 3:1–53.

⁴² Ohm's note is in the first packet, paragraph 7, p. R.15.

⁴³ Röber, "Combinationstöne und Stösse" (cit. n. 41), p. 53.

⁴⁴ Turner argues as much when he writes that "from the beginning, . . . the dispute over Ohm's law involved the problem of combination tones": Turner, "Ohm–Seebeck Dispute," p. 13.

initially an offshoot of an intended larger treatise on combination tones. It therefore seems likely that Ohm first considered using Fourier's theorem to analyze the overtone structure of primary tones in support of his case against Hällström and his hypothesis regarding the role of harmonics. While developing this approach, however, he "was suddenly assailed by a not insignificant doubt." Up to that point he had assumed that elemental tones were always, and by definition, sinusoidal. This had inspired his use of Fourier's theorem. But this basic assumption was called into question by Röber's article from 1839 and, even more forcefully, in two articles by Seebeck from 1841 and 1842.⁴⁵

DEFINING A TONE: OHM VERSUS SEEBECK

The "tone" Ohm sought to define was rather specific: it is a well-formed, harmonic tone with a clearly defined pitch—a *musical* tone. Even more specifically, the tones he discussed were those produced during Seebeck's experiments with a mechanical siren, described in the latter's article from 1841. In contrast to traditional musical instruments, a siren can sustain clearly pitched tones and is therefore very suitable for acoustical studies—especially regarding pitch. Furthermore, because its tones are not produced by continuous motion (a moving bow, a stream of air) but by discrete puffs of air blown through the holes of a rotating disc, the siren suggests that the production of musical tones is not contingent on the kind of continuity that had long been assumed to underpin all natural phenomena.⁴⁶

This is exactly what Röber argued in 1839: recent siren experiments showed that the "nature of a tone" might be defined by "the regular repetition of *any* impulse," which means that pitch is constituted solely by "the number of simple and compound impulses per unit time," regardless of their shape. In other words, the production of a pure tone does not depend on the shape of sound impulses (in modern terminology, on their waveform), but only on their repetitive, periodic nature. Seebeck's 1841 and 1842 articles subsequently proposed something even more radical: his experiments with a double (or "polyphonic") siren, using two discs with unevenly spaced holes, suggested that clearly pitched tones can also be produced by impulses that are not only arbitrarily shaped but not even repeated entirely periodically. Seebeck therefore concluded that neither the shape (waveform) nor the periodicity (or "isochronicity") of the impulses is essential for defining a tone.⁴⁷

As he confessed in the first sentence of "On the Definition of a Tone," it was only when confronted with these new findings that Ohm realized that he had taken the sinusoidal shape of sound impulses "as a self-evident fact" throughout his investigations on combination tones, thereby assuming "that a succession of stimuli on our ear which consists uninterruptedly of the

⁴⁵ Ohm, "On the Definition of a Tone" (cit. n. 1), p. 243; Röber, "Combinationstöne und Stösse" (cit. n. 41); August Seebeck, "Beobachtungen über einige Bedingungen der Entstehung von Tönen," *Ann. Phys. Chem.*, 1841, 2nd Ser., 53:417–436; and Seebeck, "Akustik," *Rep. Phys.*, 1842, 6:3–100. Most of Seebeck's first article is reproduced in the second. In 1843 Ohm first cites Röber's article from 1839, which refers to Seebeck's article "Ueber Klirröne" (1837). He subsequently refers to Seebeck's 1841 article. See Ohm, "On the Definition of a Tone," pp. 243–244. The unpublished material mentions only Seebeck's 1842 article.

⁴⁶ Steege, *Helmholtz and the Modern Listener* (cit. n. 4), p. 48. See also Myles W. Jackson, "From Scientific Instruments to Musical Instruments: The Tuning Fork, the Metronome, and the Siren," in *The Oxford Handbook of Sound Studies*, ed. Trevor Pinch and Karin Bijsterveld (Oxford: Oxford Univ. Press, 2012), pp. 201–223; Philipp von Hilgers, "Sirenen: Lösungen des Klangs vom Körper," *Philosophia Scientiae*, 2003, 7:85–114; and Alexander Rehding, "Of Sirens Old and New," in *The Oxford Handbook of Mobile Music Studies*, Vol. 2, ed. Sumanth Gopinath and Jason Stanyek, July 2014, <https://doi.org/10.1093/oxfordhb/9780199913657.013.003>.

⁴⁷ Ohm, "On the Definition of a Tone" (cit. n. 1), p. 244 (quoting Röber) (emphasis added); and Seebeck, "Beobachtungen über einige Bedingungen der Entstehung von Tönen" (cit. n. 45), pp. 421–425.

form here specified must produce the sensation of a tone.” Convinced of this assumption, as the unpublished notes show, he turned to Fourier’s theorem to assess the role of harmonic overtones in the production of higher-order combination tones. So when Röber’s and Seebeck’s articles questioned this crucial assumption and suggested that the sinusoidal shape might be, as Röber argues, “only a special case,” the whole endeavor seemed to be in jeopardy.⁴⁸

This is why Ohm set aside the work on combination tones; first he needed to show that the analytical method he had been developing actually “proved” something he assumed to be commonly understood anyway: the sinusoidal form of elemental tones and their role in defining pitch. To disprove the claim that elemental tones can be produced by series of nonperiodic *and* nonsinusoidal impulses, he introduced “the theorem of Fourier as a means of judging whether in a given impulse [a simple harmonic vibration] is contained as a real component or not.”⁴⁹ In other words, he applied Fourier analysis to show that a simple sinusoidal component (a sine wave) with a frequency corresponding to the fundamental pitch is physically present in all of Seebeck’s examples, whether simple or compound. Crucially, he did not conduct any new experiments to support this claim but relied entirely on Seebeck’s data.

The conclusion of Ohm’s analysis came down to three main points.⁵⁰ First, he argued, contra Seebeck, that elemental tones are *only* produced by periodic impulses. Second, contra Röber, he showed that such elemental tones can *only* be represented by simple harmonic motion, or what would later be called “sine waves.” Third, he argued that any other shape (or waveform) produces a compound tone. The article remained somewhat unclear, however, on the interpretation of these results beyond Seebeck’s examples, and it remained to be seen how general the application of Fourier analysis and the congruent equivalence between sinusoidal motion and elemental tones actually was.

By calling the notion that simple harmonic motion defines all musical tones the “old definition of a tone,” Ohm seemed to assume that this definition had been more or less unequivocally accepted until Seebeck’s experiments called it into question.⁵¹ Even in the early 1840s, however, the structural equivalence between elemental tones and sinusoidal motion was contested. The origins of this equivalence go back to Mersenne’s work in the first half of the seventeenth century, and the first concrete connection between vibrating strings and sinusoidal motion was made by Christiaan Huygens in 1673. Subsequently, the analysis of Brook Taylor and observations by Sauveur, both published in 1713, constituted the earliest applications of trigonometric functions to the problem.⁵² Still, for both Taylor and Sauveur, the sine function represented the movement of an idealized string (the sound source) itself and not the vibrating air (the sound) it sets in motion. A trigonometric representation of the sound wave itself only appeared in Daniel Bernoulli’s work between the 1730s and the 1750s. Bernoulli was the first to propose that the sound produced by a vibrating string might be represented by series of superpositioned sinusoidal frequencies—or what would later be known as a Fourier series.⁵³

⁴⁸ Ohm, “On the Definition of a Tone,” pp. 243, 242 (quoting Röber).

⁴⁹ *Ibid.*, p. 246. Maley translates this as “either entirely pure in itself or this form must at least be separable, as a real constituent part from each impulse”: Maley, *Theory of Beats and Combination Tones* (cit. n. 24), p. 113.

⁵⁰ Muzzolini, “Genealogie der Klangfarbe” (cit. n. 7), p. 43.

⁵¹ Ohm, “On the Definition of a Tone” (cit. n. 1), p. 244.

⁵² Darrigol, “Acoustic Origins of Harmonic Analysis” (cit. n. 6), p. 351. On the history of the sine function see Muzzolini, “Genealogie der Klangfarbe” (cit. n. 7), pp. 135–138; Dirk Jan Struik, ed., *A Source Book in Mathematics, 1200–1800* (Princeton, N.J.: Princeton Univ. Press, 1986); and V. Frederick Riekey, “Etymology of the Word ‘Sine,’” 1996, <http://fredrickey.info/hm/Calc/Notes/EtymologySine.pdf> (accessed 15 Apr. 2018).

⁵³ Muzzolini, “Genealogie der Klangfarbe,” p. 191.

This idea, in turn, was heavily contested by Euler, d'Alembert, and Lagrange, who treated the problem of the vibrating string primarily from an idealized mathematical perspective.⁵⁴ Contrary to Euler, who believed that the superposition principle only applied to special cases, Bernoulli believed that his trigonometric analysis was a “general, physically meaningful way of describing any vibration,” because infinite sums of “simple modes” could represent any arbitrarily shaped waveform. Following Bernoulli's analysis, then, simple sinusoidal motion is considered elementary. The validity of his “law of small oscillations,” however, was not settled until Fourier's theorem in 1822 and Dirichlet's proof in 1829. Only at this point was it established that every complex function could indeed be analyzed into separate “simple modes,” making Fourier's method a powerful analytical tool to decompose all kinds of seemingly arbitrary curves into series of regular components.⁵⁵ Its first use in this manner was Ohm's analysis of 1843. The question of how to interpret the resulting representation of harmonic overtone series, however, became the focal point of yet another intense dispute.

As “On the Definition of a Tone” was a direct response to Seebeck, it was Seebeck who replied four months later with “On the Siren.” After praising Ohm for his interesting contribution, Seebeck wrote that his disagreement with his colleague's findings concerned the “idea one should have of the specific waveform of different sounds.”⁵⁶ In short: Is an elementary tone always sinusoidal, or not? He also pointed to a significant mathematical error, which had led Ohm to conclude that, in cases where the duration of the siren impulse is as long as the time between two puffs of air, the amplitude of the fundamental frequency seems to be infinite. Clearly, this is physically impossible, but Ohm argued that the result could be a limiting case, because it also explains why the fundamental tone is usually heard much louder relative to the accompanying harmonics. Seebeck corrected the mistake but noted that the analysis was still not able to predict correctly the relative intensity of the harmonics as they are perceived by the ear.⁵⁷

In addition, Seebeck's hesitations were caused by an incorrect assumption on Ohm's part. On the basis of Seebeck's initial report, Ohm assumed that a regular siren (with one disc and evenly spaced holes) produces simple, sinusoidally shaped tones, whereas harmonic overtones only accompany tones produced by the double siren with unevenly spaced holes. If regular siren tones were indeed sinusoidal, as Ohm assumed, it made perfect sense to use Fourier's trigonometric analysis (based on series of such sinusoids) to assess the compound tones of a double siren. In his response, however, Seebeck admitted that he had failed to report earlier

⁵⁴ Darrigol, “Acoustic Origins of Harmonic Analysis” (cit. n. 6), p. 401. Darrigol provides a comprehensive summary of the debate, to which Euler, d'Alembert, and Lagrange each made valuable contributions. None of them, however, considered sinusoidal vibration as the only elementary shape, and all three “denied any physical meaning to harmonic analysis.” D'Alembert did not believe in the physical existence of harmonic partials at all and attributed audible harmonics to causes like acoustic resonance. Euler was the first to apply trigonometric series to the problem of the vibrating string successfully but believed that his solution only applied to certain restricted cases. Lagrange, finally, regarded the idea of the coexistence of small oscillations as a “mathematical fiction.” *Ibid.*, pp. 345, 396–401. See also Clifford Truesdell, “The Rational Mechanics of Flexible or Elastic Bodies, 1638–1788,” in *Leonhardi Euleri Opera Omnia, Vol. X and XI* (Zurich: Orell Füssli, 1960), pp. ix–cxxx.

⁵⁵ Darrigol, “Acoustic Origins of Harmonic Analysis,” p. 401; Muzzulini, “Genealogie der Klangfarbe” (cit. n. 7), p. 341; and Wise, “What's in a Line?” (cit. n. 21), p. 82.

⁵⁶ August Seebeck, “Ueber die Sirene,” *Ann. Phys. Chem.*, 1843, 2nd Ser., 60:449–481, on p. 449. Detailed assessments of the dispute are found in Turner, “Ohm–Seebeck Dispute”; Kursell, *Epistemologie des Hörens* (cit. n. 4); Maley, *Theory of Beats and Combination Tones* (cit. n. 24); Steege, *Helmholtz and the Modern Listener* (cit. n. 4); and Stephan Vogel, “Sensation of Tone, Perception of Sound, and Empiricism: Helmholtz's Physiological Acoustics,” in *Hermann von Helmholtz and the Foundations of Nineteenth-Century Science*, ed. David Cahan (Berkeley: Univ. California Press, 1994), pp. 259–287.

⁵⁷ Vogel, “Sensation of Tone, Perception of Sound, and Empiricism,” p. 264; and Turner, “Ohm–Seebeck Dispute,” pp. 6–7.

that regular siren tones also include “one or more tones of the harmonic series accompanying the fundamental tone” and are thus not sinusoidal at all.⁵⁸

Although Seebeck acknowledges that this fact could actually support Ohm’s definition of a tone, he also notes that it still does not explain the discrepancy between the perceived intensity of the harmonics and Ohm’s analysis: if the analysis were correct, one would expect the harmonics to sound much louder. However, even when Seebeck revisited his experiments using Ohm’s analysis but without the latter’s mathematical error and erroneous assumption, it predicted the harmonics to have a far greater intensity than perceived by the ear. As a solution, Seebeck suggested that the higher harmonics together might produce some kind of combination tone that reinforces the fundamental and makes them less audible as such, but, against Ohm’s definition, this would mean that the pitch of a compound tone is not always produced by a fundamental sinusoidal vibration.⁵⁹ And so he remained unconvinced.

To conclude his case against Ohm, Seebeck raised one final objection, using yet another siren experiment. This final remark provides perhaps the best example of the tension between Ohm’s mathematical analysis and Seebeck’s empirical approach. When using a siren with not two but three rotating discs, Seebeck reported, he clearly heard a fundamental pitch that sounded much louder than Ohm’s theory would predict. Even more important, the actual frequency corresponding to this pitch did not seem to be present at all. By showing that a clear pitch can also be produced by the harmonics alone, without a fundamental, sinusoidal frequency being present, this example problematized Ohm’s definition on two counts: the strength of the harmonics and the sinusoidal shape of the fundamental frequency.⁶⁰ Unbeknownst to Seebeck, this three-disc siren experiment exactly simulated a phenomenon later dubbed the “missing fundamental.” It often occurs with instruments with more complex and mostly nonharmonic overtone structures, like bells or vibrating plates.⁶¹

All in all, Seebeck’s response seemed to undermine much of Ohm’s “new” “old definition” of a tone, not to mention his intellectual credibility as an acoustician. Ohm, however, remained undeterred. As is often noted, in light of subsequent developments, the most significant aspect of his response to Seebeck in 1844 (which would turn out to be his final publication on acoustics) was a speculative solution to the former’s objections regarding the discrepancy between the predicted and the perceived loudness of harmonic overtones. Although he admits being unable to offer a fully satisfactory rejoinder, Ohm suggests that our ears might be misled or may become accustomed to attributing “a stronger strength to the fundamental . . . and a weaker strength to its subsidiary tones.”⁶² This attribution of the discrepancy between theory and observation

⁵⁸ Hilgers, “Sirenen” (cit. n. 46), p. 90; and Seebeck, “Ueber die Sirene” (cit. n. 56), p. 453.

⁵⁹ Turner, “Ohm–Seebeck Dispute,” p. 8; and Seebeck, “Ueber die Sirene,” p. 474.

⁶⁰ Seebeck, “Ueber die Sirene,” pp. 466, 479.

⁶¹ Bernhard Siebert, “Mineral Sound or Missing Fundamental: Cultural History as Signal Analysis,” *Osiris*, 2018, N.S., 28:105–118, esp. p. 111. Cheveigné writes that J. F. Schouten in 1938 “confirmed Seebeck’s observation that the fundamental partial is dispensable,” calling the phenomenon “residue pitch.” More recent studies, however, showed that this concept “is no longer useful and the term ‘residue pitch’ should be avoided.” See Cheveigné, “Pitch Perception Models” (cit. n. 3), pp. 192–193.

⁶² Ohm, “Noch ein Paar Worte über die Definition des Tones” (cit. n. 10), p. 15. Ohm’s only reference to (somewhat) empirical research concerns this tentative suggestion that the ear plays tricks on the listener: Ohm called on a violinist friend to help assess whether “when playing a tone together with its octave and suddenly leaving out the lower tone, it appears as if the remaining higher tone becomes stronger,” thus proving that the higher tone is heard less forcefully when combined with a lower fundamental. *Ibid.*, p. 16. Füchtbauer identifies this friend as Dr. Kellermann, a colleague and former student of Ohm: Füchtbauer, *Georg Simon Ohm* (cit. n. 29), p. 139. However, some unnumbered pages in the fourth packet of the archival material, containing a few musical bars with commentary (possibly the report of Kellermann), suggest that the experiment in fact concerned combination tones. Besides this attempt, Ohm’s student Friedrich Mann recalls another experiment: with the help of students “who claimed to have good musical hearing” and “a very primitive model of an organ,” Ohm “staged a concert that must have been

to something like an acoustic illusion is the only point in the entire dispute where the role of the faculty of hearing itself is explicitly considered; and whereas Seebeck disregarded the possibility of an acoustic illusion entirely, Ohm did not pursue the idea either.

Although, in hindsight, Ohm's remark opened a trajectory toward a more psychoacoustic approach to the problem, his dispute with Seebeck remained almost entirely within the realm of mathematics and physics.⁶³ Whereas Ohm reduced "tone" to a sinusoidally shaped fundamental (a sine wave), Seebeck questioned this reduction both to the fundamental and to sinusoidal motion, because, as he argues, even in the case of a vibrating string such simple harmonic motion is a mathematical abstraction and at best an approximation of the physical case. Furthermore, he argues, although Ohm's analysis accounts for pitch and loudness, it "does not allow for any diversity of sounds"—that is, for tone color or timbre. Seebeck therefore calls Ohm's hypothesis (that "a sinusoidally shaped fundamental frequency—and this frequency alone—defines the pitch of a tone") the "narrow assumption" or "narrow form" of a possible definition. The "broader assumption" or "broader form," by contrast, defines a tone on the basis of *any* periodic repetition of impulses, whether simple sinusoids or arbitrarily shaped compound waveforms.⁶⁴

This uncertainty regarding the status of overtones is also reflected by changes in terminology. In 1841 and 1842 Seebeck calls the puffs of air that produce the siren sound "impulses [*Impulse*]," whereas Ohm's "On the Definition of a Tone" does not refer to the puffs themselves but to the physical (not psychological) "impressions [*Eindrücke*]" they produce. Seebeck initially used that term more metaphorically, to describe the listener's "impression of a clear pitch," but adopts Ohm's more literal use in his response, to denote the impression of sound on the eardrum. Ohm, in turn, also calls the individual terms in the Fourier series (denoting different partials) "first," "second," and "third impression" and, on one occasion, uses the more general term "partial impression [*Partialeindruck*]." ⁶⁵ In the unpublished manuscript, he also uses "compound impression [*Gesamteindruck*]" and "overall impression [*Totaleindruck*]." Seebeck's response, however, resorts to more musical terms: "main tone [*Hauptton*]" and "subsidiary tones [*Beitöne*]." Ohm, who did not use such musical terminology in the 1843 article, adopts these terms in his response.⁶⁶

These linguistic shifts might seem superficial, but they reflect profound phenomenological and epistemological uncertainties. Whereas Seebeck very much relied on the empirical practice of interpreting qualitative results in preference to mathematical explanations and treated the ear, as Benjamin Steege puts it, as "transparent to knowledge," Ohm sought a general mathematical theory, as he had done in *The Galvanic Circuit*. Like Bernoulli's model of the vibrating string a

most interesting in light of the physics, but otherwise a cacophony to hear." See Mann, *Georg Simon Ohm* (cit. n. 29), pp. 46–47. As Mann's final school year was 1842–1843, the experiments cannot have been performed in response to Seebeck's objections, because these were not published until December 1843. See *Jahresbericht über die technische und landwirthschaftlichen Lehranstalten in Nürnberg bekannt gemacht am Schlusse des Schuljahres 1842/43* (Nuremberg: Campe, 1843), p. 40, <https://www.th-nuernberg.de/einrichtungen-gesamt/zentrale-cinrichtungen/bibliothek/elektronische-angebote/historische-quellen-am-ohm> (accessed 2 July 2019).

⁶³ Ohm's remark, Turner notes, shows "how closely the Ohm–Seebeck controversy was bound up in the difficult issues concerning the physiological, psychological, and even philosophical dimensions of sensory perception": Turner, "Ohm–Seebeck Dispute," p. 10.

⁶⁴ Seebeck, "Ueber die Definition des Tones" (cit. n. 35), pp. 361, 364, 353–354.

⁶⁵ Seebeck, "Beobachtungen über einige Bedingungen der Entstehung von Tönen" (cit. n. 45), p. 423; and Ohm, "Über die Definition des Tones" (cit. n. 1), p. 520.

⁶⁶ Helmholtz cleared up this terminological confusion. He used "musical tone [*Klang*]" to refer to any repetitive compound sound and "simple tone [*Ton*]" to describe Ohm's sinusoidal components. For Helmholtz, then, a combination of various sounds (*Zusammenklang*) can contain multiple musical tones (*Klänge*), which in turn consist of series of simple tones (*Töne*). Helmholtz, *On the Sensations of Tone* (cit. n. 2), pp. 35–36. For the original see Hermann von Helmholtz, *Die Lehre von den Tonempfindungen als Physiologische Grundlage für die Theorie der Musik*, 6th ed. (Braunschweig: Vieweg, 1913), p. 39.

century earlier, Ohm's analysis focused not on the sound source (puffs of air) but on the physical impression (the sound wave) itself. The changing terminology shows, however, that neither of the two knew exactly how to interpret the relation between Seebeck's experimental results and Ohm's mathematical analysis—or its relevance (or irrelevance) for music theory. Whereas Seebeck asked how we “can . . . determine what belongs to a tone, if not with the ear,” Ohm treated mathematical analysis as an experimental tool in and of itself, trusting his mathematical instinct even after Seebeck's response seriously questioned its credibility.⁶⁷ Ultimately, neither of them recognized the profound epistemological consequences of their dispute. These would only come to light with Helmholtz's intervention more than a decade later.

THE UNMUSICAL EAR

By the end of the 1840s, the dispute between Ohm and Seebeck had drawn to an unsatisfactory close. Ohm absented himself from the argument in 1844 and refrained from publishing on acoustics any further. Seebeck passed away in 1849. In hindsight, the tension between Ohm's analysis and Seebeck's experimental observations can be attributed at least in part to Ohm's position as an early adopter of the more analytical scientific paradigm that emerged in Germany in the third and fourth decades of the nineteenth century. Like Fourier in France earlier in the century, Ohm was an exponent of a broader intellectual shift during which the epistemological premises of and disciplinary boundaries between pure mathematics, experimental physics, and mathematical physics were being redefined. Against established experimental practice in early nineteenth-century Germany, Ohm's work of the 1820s explicitly followed Fourier's analytical approach in its aim to set forth a general mathematical theory of the electrical circuit. Still, whereas his quantitative experimental work anticipated the precise methods that would define physical research in the following decades, his separation of experimental data and mathematical analysis did not.⁶⁸

Similarly, although the acoustical articles of the 1840s constitute a shift toward more extensive use of mathematical models, in contrast to his earlier work, the almost complete absence of experimental verification shows that Ohm had not yet fully taken on the younger generation's rigorous ethos with respect to quantitative experimental research. Furthermore, whereas the didactic “transition from intuiting to thinking” outlined in his textbook of 1817 pointed toward a more hypothetico-deductive understanding of the relation between empirical experiment and mathematical analysis, as late as 1842 Ohm also expressed the fear that too much emphasis on mathematical analysis could rob experimental work “of its original simplicity.”⁶⁹ Such fear notwithstanding, the acoustic articles, like *The Galvanic Circuit*, not only applied Fourier's theorem but also followed his lead in privileging mathematical models over empirical observation, albeit this time relying almost completely on experimental data provided by others.

⁶⁷ Steege, *Helmholtz and the Modern Listener* (cit. n. 4), p. 51; and Seebeck, “Ueber die Sirene” (cit. n. 56), p. 361.

⁶⁸ For an assessment of Ohm's experimental practice in the 1820s see John L. McKnight, “Laboratory Notebooks of G. S. Ohm: A Case Study in Experimental Method,” *Amer. J. Phys.*, 1967, 35:110–114; and Michael Heidelberger, “Some Patterns of Change in the Baconian Sciences of the Early Nineteenth Century Germany,” in *Epistemological and Social Problems of the Sciences in the Early Nineteenth Century*, ed. H. N. Jahnke and M. Otte (Dordrecht: Reidel, 1979), pp. 3–18, esp. p. 14.

⁶⁹ Ohm, *Grundlinien zu einer zweckmäßigen Behandlung der Geometrie als höheren Bildungsmittel an vorbereitende Lehranstalten* (cit. n. 12), p. v; and Jungnickel and McCormach, *Intellectual Mastery of Nature* (cit. n. 15), p. 119 (quoting Ohm). The original comment is from a footnote by Ohm in Paul Wolfgang Haecker, “Versuche über das Tragvermögen hufeisenförmiger Magnete und über die Schwingungsdauer geradliniger Magnetstabe,” *Ann. Phys. Chem.*, 1842, 2nd Ser., 57:321–345, on p. 322. Regarding the “concretizing” drive toward explanatory simplicity note also Ohm's statement in “On the Definition of a Tone” that “no other causes should be assumed than are both necessary and sufficient”: Ohm, *On the Definition of a Tone* (cit. n. 1), p. 245. Füchtbauer calls this Ohm's “scientific article of faith [*naturwissenschaftliches Glaubensbekenntnis*]”: Füchtbauer, *Georg Simon Ohm* (cit. n. 29), p. 191.

As a result, Ohm's conclusions remained torn between the concretizing tendencies of his youth and the abstracting approaches that would follow. On the one hand, his approach seems to suggest that mathematical analysis can access a physical domain beyond that which is directly accessible to sensory perception. On the other hand, he did not conduct any original experiments and seemed to assume that mathematics could unproblematically uncover the "simple laws" of nature. Whereas Seebeck could not accept a hypothesis that went against the evidence of his own two ears, Ohm remained convinced that his analysis was correct, even in the absence of empirical proof. Only the tentative suggestion that the ear might play tricks on the hearer fell outside this discursive comfort zone; and only with Helmholtz did it become clear that the analysis did indeed concern not only objective physical properties but also the way in which we perceive them.⁷⁰

Early in his investigations, Helmholtz verified Ohm's definition by putting the latter's suggestion from 1839 into practice: he compared the combination tones of sounds with no discernible harmonic overtones (those made by tuning forks) with those with clearly audible harmonic overtones (those made by vibrating strings). Using his acoustic resonators to "hear out" individual partials, he subsequently verified the harmonics of different sounds and at the same time proved the ear's ability to "hear them out," even if they normally go unnoticed. On the basis of these findings, Helmholtz forged a connection between mathematical theory and acoustic perception and reframed Ohm's mathematical analysis as psychophysical or psychoacoustic theory.⁷¹ He thereby turned Ohm's speculative aside about the role of the ear into the hypothesis that the inner ear *itself* functions as a Fourier analyzer, decomposing sounds into elemental sinusoidal components. What began in the mathematical realm was being converted into a putatively physiological order. By raising Ohm's initially controversial analysis to the status of a proper "acoustic law," Helmholtz consolidated the idea of simple sinusoidal motion as the "basic element" of sound.⁷²

But instead of focusing on this frequently cited speculation about our possibly deceptive ears (which is often, somewhat anachronistically, read as a prefiguration of Helmholtz's resonance theory), I want to highlight another brief remark in Ohm's response to Seebeck in 1844. This remark allows for a more nuanced reading of Ohm's work on its own terms and clarifies the extent to which subsequent conceptualizations of sound, from Helmholtz up to the present day, retained central assumptions that already informed Ohm's analysis. After briefly summarizing his own article and Seebeck's response, Ohm explains that he will "review the weight" of the objections raised against his analysis "at least through close consideration of the object of study"; but, he confesses, he cannot verify his findings empirically because "nature has altogether denied

⁷⁰ Turner, "Ohm–Seebeck Dispute," p. 7.

⁷¹ Helmholtz, "Ueber Combinationstöne" (cit. n. 2), pp. 501–502 ff.; and Helmholtz, *On the Sensations of Tone* (cit. n. 2), pp. 84–100. It should be noted that, as Alexandra Hui remarks, "Helmholtz would not have called himself a psychophysicist or his project psychophysical. . . . For him, a study of sound sensation was done through an examination of sound as an external, physical object": Alexandra Hui, *The Psychophysical Ear: Musical Experiments, Experimental Sounds, 1840–1910* (Cambridge, Mass.: MIT Press, 2013), p. 58.

⁷² Crucially, in hindsight, Seebeck's objections were not unfounded. Although the analytical representation of sound as series of periodic sine waves is very effective, Ohm's acoustic law alone does not suffice to explain pitch perception, because it only applies to periodic sounds. Although the integral solution to Fourier's theorem (see note 5, above) can be applied to nonperiodic sounds as well, it also disregards temporal development. Seebeck's views are now considered an early example of the "time theory" of hearing, or the theory of "periodicity pitch," which is generally thought to complement Helmholtz's resonance or "place theory." See Cheveigné, "Pitch Perception Models" (cit. n. 3), pp. 180–183; and E. de Boer, "On the 'Residue' and Auditory Pitch Perception," in *Handbook of Sensory Physiology*, Vol. 3, ed. Wolf D. Keidel and William D. Neff (New York: Springer, 1976), pp. 479–584.

me a musical ear.”⁷³ This suggests that his “unmusical ear” led him to pursue his strictly mathematical definition of a tone without conducting serious experimental work.

Admittedly, it cannot be ruled out that Ohm’s self-confessed “unmusicality” is a matter of false modesty triggered by the embarrassment he suffered at Seebeck’s response. The available evidence suggests otherwise, however. It is not unlikely that Ohm picked up some knowledge of music theory during his time as a student at the Erlangen *Gymnasium* and university and as a teacher in Cologne and Nuremberg. As his writings attest, he knew the basic principles of harmony and was familiar with the mechanics of sound as they had been established by the 1830s. On the other hand, there is no evidence at all of any further training or interest in music, nor for a more specific interest in acoustics apart from the three articles of 1839, 1843, and 1844. Moreover, in 1890 Ohm’s former student Friedrich Mann recalled that, at least by the early 1840s, his teacher “had completely lost the ability to discern musical tones.”⁷⁴ This suggests that there might even have been a physiological cause for Ohm’s “unmusical ear.” I therefore think it fair to assume that, at the very least, Ohm felt uncomfortable and ill equipped with regard to music.

In contrast to the more mathematically informed theories of the eighteenth century, the music theoretical discourse of the early and mid-nineteenth century generally considered a “tone” to be a singular unit with a uniform pitch.⁷⁵ Correspondingly, a “musical ear” was defined primarily in relation to the rules of harmony and issues of consonance and dissonance. According to Johann Ernst Häuser’s *Musikalisches Lexikon* (1833), for example, a musical ear can determine “correct and incorrect connections between tones in harmony or in the melody, as soon as one hears them.”⁷⁶ Whether this capacity is a matter of nurture or nature remained to be seen. In Britain, William S. Porter’s *Musical Cyclopaedia* (1834) considered it “chiefly the result of cultivation,” whereas the natural philosopher William Whewell argued in 1840 that such an ear for music “is nearly universal among men.” Taking a kind of middle position, August Gathy’s *Musikalisches Conversations-Lexikon* (1840) distinguishes between a “musical ear [*musikalisches Gehör*],” designating the capacity to perceive musical tones and relations as such, and “musical feeling [*musikalisches Gefühl*],” or the ability “to process the higher significance of music.”⁷⁷ As the flip side of these musical ears, Ohm’s unmusical ear therefore designates an inability to perceive harmonic and melodic pitch relations correctly, let alone process their musical significance. In modern parlance: tone deafness.

It is precisely the fact that someone who was apparently tone-deaf made one of the most important contributions to modern acoustics that makes Ohm’s remark on his unmusical ears

⁷³ Ohm, “Noch ein Paar Worte über die Definition des Tones” (cit. n. 10), p. 7.

⁷⁴ Mann, *Georg Simon Ohm* (cit. n. 29), p. 46. There are no further sources to corroborate this account, and it should be noted that it was written more than forty years after the fact. Ohm uses some basic musical terms in Ohm, “Bemerkungen über Combinationstöne und Stösse” (cit. n. 24). There are a few pages of musical script among his notes in the archive, although these could also be by Ohm’s musical friend (see note 62, above).

⁷⁵ Kursell, *Epistemologie des Hörens* (cit. n. 4), p. 287. Kursell cites Gottfried Weber’s *Theorie der Tonsetzkunst zum Selbstunterricht* (1817) and Adolf Bernhard Marx’s *Musiklehre* (1839).

⁷⁶ Johann Ernst Häuser, *Musikalisches Lexicon* (Meißen: F. W. Goedsche, 1833), p. 168. Similarly, William Gardiner writes that “a musical ear” is acquired “by pursuing a course of study in harmony”: William Gardiner, *The Music of Nature* (1832; Cambridge: Cambridge Univ. Press, 2014), <https://doi.org/10.1017/CBO9780511694806>, p. 9.

⁷⁷ William S. Porter, *The Musical Cyclopaedia; or, The Principles of Music* (Boston: James Loring, 1834), p. 136; William Whewell, *Philosophy of the Inductive Sciences*, Vol. 1 (1840) (Cambridge: Cambridge Univ. Press, 2014), <https://doi.org/10.1017/CBO9781139644662>, p. 310; and August Gathy, *Musikalisches Conversations-Lexikon: Encyclopädie der Gesammten Musikwissenschaft* (Hamburg: E. W. Niemeyer, 1840), p. 319. Gustav Schilling adds that “only by practicing the production of pure musical tones oneself, and not by merely hearing such tones, can the ear be trained”: Gustav Schilling, *Musikalische Dynamik, oder die Lehre vom Vortrage in der Musik* (Kassel: J. C. Krieger, 1843), p. 159.

so significant. First, it supports my argument that Ohm was not attracted to the problem of combination tones or the definition of a tone from a musical or sonic perspective but, instead, because of the potential for mathematical analysis. Second, and more important, it means that, against a dominant music theoretical discourse that was largely unconcerned with physics or mathematics, his approach did not involve music theoretical considerations at all.⁷⁸ Instead, by turning to Fourier analysis, and thus essentially picking up the intellectual lineage of Bernoulli's work on the vibrating string in the mid-eighteenth century, Ohm returned to a pre-nineteenth-century approach in which music and mathematics had been more closely aligned. Because he could not use a "natural" or supposedly innate capacity to perceive tones and tonal relations in a musical sense, he turned to the abstracting science of mathematical analysis. He thereby essentially transformed his unmusical ear into an epistemological tool that highlights what is, in the context of Western music theory, the decidedly "unmusical" nature of physical sound and anticipates the changing role of the senses—aided by or extended through mathematics and technology—in physical research.

In the case of combination tones, most eighteenth-century studies treated the problem from a primarily musical perspective, but the analysis of Young, Weber, Hällström, and ultimately Ohm gradually adopted a more mathematical approach. This had been anticipated by Lagrange's analysis in 1759, which, not coincidentally, also marked the first time that combination tones were linked to the problem of the vibrating string. The debate on the vibrating string, in turn, occurred at the intersection of, on the one hand, the seventeenth- and eighteenth-century paradigm that still considered music as a science closely related to mathematics and physics with, on the other hand, the rise of late eighteenth- and nineteenth-century aesthetic idealism, through which music and the sciences became increasingly separate.⁷⁹ Eighteenth-century mechanical analyses of vibrating strings regularly intersected with questions of musical harmony, theories of consonance, and issues of instrument tuning. In the first decades of the nineteenth century, however, the mathematical models that had been developed in this context were far removed from sound, not to mention music: they were taken up in Fourier's *Analytical Theory of Heat* and in purely mathematical terms by Dirichlet and others.

This shift from music to mathematics also marks Ohm's articles between 1839 and 1844. Whereas his assessment of Hällström's theory of combination tones still explicitly mentions musical pitches and intervals ("F-sharp," "A," "a third"), such terminology is often replaced by or supplemented with frequency numbers and mathematical ratios in the unpublished manuscript. The two articles on the definition of a tone, finally, do not mention pitches or intervals at all. They only deal with mathematical ratios. Keeping in mind that, as Friedrich Kittler remarks, when "musicians read the sign, mathematicians . . . read the number," these changes show Ohm's increasing confidence in dispensing with music theory altogether in favor of mathematical analysis.⁸⁰ With his application of Fourier's theorem, first to the problem of combination tones and subsequently to Seebeck's siren tones, the separation of music theory and

⁷⁸ Adolph Marx, for instance, writes that the relation between acoustics and "music as an art" is only superficial, because "the activity and enjoyment of the soul in the perception of music is fundamentally different from the apprehension of a sensory event (a tone) by the mind": Adolph Bernhard Marx, *Die alte Musiklehre im Streit mit unsere Zeit* (Leipzig: Breitkopf und Härtel, 1841), p. 149.

⁷⁹ See, e.g., Emily I. Dolan, "Music as an Object of Natural History," in *Sound Knowledge: Music and Science in London, 1789–1851*, ed. James Q. Davies and Ellen Lockhart (Chicago: Univ. Chicago Press, 2016), pp. 27–46.

⁸⁰ Ohm, "Bemerkungen über Combinationstöne und Stösse" (cit. n. 24), p. 264; and Friedrich Kittler, "Der lange Weg zur Compact Disc," in *Amor vincit omnia: Karajan, Monteverdi und die Entwicklung der neuen Medien*, ed. Sigrít Fleiss and Ina Gayed (Vienna: Herbert von Karajan Centrum, 2000), pp. 215–231, on p. 220.

physical acoustics—an increasingly strained marriage at least since Sauveur’s coinage of the word “acoustics” in the early eighteenth century—achieved a new momentum.

Although both Fourier analysis and Western musical notation are idealized symbolic representations of acoustic phenomena, only an approach that leaves the signifying logic of music notation and its age-old theoretical framework behind can begin to deal with questions of sound beyond a (Western) musical perspective. Seen like this, it becomes clear that it was not despite but precisely because he could not use his ears in a “musical” way that Ohm made his groundbreaking contribution to acoustical theory.⁸¹ Precisely because Fourier analysis assesses the physical nature of sound without regard for musical intervals, melodies, or harmonies, Ohm could let go of music altogether and develop a new definition of a tone. Although many elements of this definition (the sinusoidal shape of simple tones, the basic principles of superposition, the periodicity of uniformly pitched sounds) had been established in one way or another for decades or even centuries, Ohm was the first to bring them together in a concise mathematical theory.

How far removed this was from music theoretical and aesthetic discourse at the time is nicely illustrated by Eduard Hanslick’s view on the relationship between music and mathematics. In *On the Musically Beautiful*, published in the year of Ohm’s death, 1854, the Viennese music critic writes: “Although mathematics furnishes an indispensable key for researching the physical dimension of music, its significance for the completed musical work should not be overestimated. . . . Mathematics regulates merely the elemental material capable of intellectual treatment, and operates concealed in the simplest relationships. However, the musical idea emerges without it.” For Hanslick, then, the “musical idea” lies beyond the “elemental,” physical material (sound) that can be analyzed by mathematics. It can only be uncovered through a thematic analysis of the composition. Around the same time, Helmholtz began developing his resonance theory of hearing to construe a link between Ohm’s mathematical acoustics and Hanslick’s musical aesthetics and find a way from mathematically regulated “elemental material” back to questions of musical beauty.⁸² By integrating Ohm’s mathematical analysis in his psychoacoustic theory of hearing, he bridged the distance between what the analysis predicts and what the ear perceives, or between “unmusical” and “musical” ears. As the title of his book reveals, Helmholtz’s work was deeply motivated by his musical interest. Before the musically skilled and knowledgeable Helmholtz could bridge the gap between physical acoustics and musical aesthetics, however, both sound and hearing had to be redefined by Ohm’s unmusical ears.

Unhindered by the culturally ingrained framework of Western music theory, Ohm’s analysis constitutes an important shift in perspective, as illustrated by the development of the concept of the sine wave. From a theoretical perspective, a sine wave is not a sound, nor is it a wave: it is the graphical representation of the mathematical function for simple harmonic motion. This concept of simple harmonic motion moved from its mathematical origins in the sine function to a mathematical model representing a vibrating string, culminating in Bernoulli’s theory of superpositioned “simple modes.” Bernoulli’s mathematical theory was refined and perfected to apply to heat propagation, representing what Fourier called “simple states.”⁸³

⁸¹ See also Hilgers, “Sirenen” (cit. n. 46), p. 103.

⁸² Eduard Hanslick, *On the Musically Beautiful* (1854), trans. Lee Rothfarb and Christoph Landerer (Oxford: Oxford Univ. Press, 2018), pp. 58–59. On Helmholtz and Hanslick see Steege, *Helmholtz and the Modern Listener* (cit. n. 4), p. 160 ff.

⁸³ As Robert Friedman writes, “Fourier believes that the individual terms of the series . . . correspond to actual physical events,” as each of the “simple states” “exists independently but combines with the others to form what we perceive as the propagation of heat in solid bodies”: Robert Marc Friedman, “The Creation of a New Science: Joseph Fourier’s Analytical Theory of Heat,” *Hist. Stud. Phys. Sci.*, 1977, 8:73–99, on p. 95.

Subsequently, with Ohm's application of Fourier analysis, these "simple states" became "partial impressions," rendering the inner structure of sound waves comprehensible, analyzable, and graphically visible, albeit by privileging the representation of spectral clarity over temporal development. Only with Helmholtz's empirical work did these "partial impressions" become "simple tones" and the sine *function* truly become a sine *wave*: an elementary waveform considered to be an independent sound in and of itself.⁸⁴

Helmholtz's "tuning-fork synthesizer" and acoustic resonators, used to provide experimental proof of Ohm's mathematical analysis by "hearing out" approximate sine waves and combining them into compound sounds, constituted a technological implementation of his newly conceived model of the operations of the inner ear, which was, in turn, based in part on Ohm's acoustic law.⁸⁵ The latter's "unmusical ear" thus became a model for the scientific scrutiny of sound. Whereas "it had been previously impossible to conduct" such detailed sound experiments "except by trained musical ears, and much strained attention properly assisted," Helmholtz writes, with these instruments "any one, even if he has no ear for music or is quite unpractised in detecting musical sounds, is put in a condition to pick the required simple tone, even if comparatively faint, from out of a great number of others."⁸⁶

In short, with proper training and the assistance of specialized technology, any ear, regardless of its capacity for musical analysis, can now assess questions of sound. Between Ohm and Helmholtz, then, the role of the ear was reconceptualized: it no longer had to be musical to be able to analyze sound. Indeed, it was no longer the "musical ear" that required extensive training to deal with matters of sound. Instead, one had to *unlearn* or get beyond the supposedly innate conditioning of musical ears, which hear compound sound as single pitches, and focus, with Häuser, on the "correct and incorrect connections between tones in harmony or in the melody." Whereas a musical ear hears sound *as music* (or what Hanslick calls the thematic "musical idea" beyond the physical materiality of sound), the unmusical ear—assisted by mathematical analysis (Ohm) and scientific instruments (Helmholtz)—turns this subjective faculty of hearing into an objective tool for experimental observation that can engage with the "elemental material" of sound *as sound*.

In the introduction to his *Analytical Theory of Heat*, Fourier already noted that the capabilities of mathematical analysis would surpass those of our biological senses.⁸⁷ Helmholtz's "reconciliation" of mathematical physics and musical aesthetics notwithstanding, by implementing Ohm's acoustical law his research further consolidated the analytical, unmusical ear as part of the basic fabric of our concepts of sound and hearing.⁸⁸ Ohm therefore stood at the center of an intellectual transformation during which French mathematical physics collided with the empirical traditions of Germany and Britain, leading toward the "abstracting" approaches of researchers like Helmholtz. The figure of the unmusical ear, in turn, exemplifies the ways

⁸⁴ Julia Kursell, "Experiments on Tone Color in Music and Acoustics: Helmholtz, Schoenberg, and *Klangfarbenmelodie*," *Osis*, 2013, N.S., 28:191–211, on p. 192.

⁸⁵ Another inspiration for this model was Alfonso Corti's discovery in 1851 of the organ named after him: each tiny hair cell on the "organ of Corti," Helmholtz speculated, resonates with a different frequency. To Julia Kursell's observation that this shows how "the physiology of the ear . . . can even provide a model for physics," I would therefore add that this goes both ways: the physiology of the inner ear provided a model for understanding the physical composition of sound; but at the same time the idea that the ear performs a Fourier analysis also served as a model for representing and understanding the physiology of the ear. See Kursell, *Epistemologie des Hörens* (cit. n. 4), p. 43.

⁸⁶ Helmholtz, *On the Sensations of Tone* (cit. n. 2), p. 69.

⁸⁷ Fourier, *Analytical Theory of Heat* (cit. n. 5), p. 24.

⁸⁸ As Bernhard Siegert writes: "According to Helmholtz, the ear is nothing more than an analogue computer, and listening to sounds and noises is nothing but real-time Fourier analysis." See Bernhard Siegert, *Passage des Digitalen: Zeichenpraktiken der Neuzzeitlichen Wissenschaften 1500–1900* (Berlin: Brinkmann & Bose, 2003), p. 368.

in which this transformation redefined the role of the senses in scientific research. Disciplined and trained to process physical sense data beyond culturally engrained frameworks, the unmusical ear represents an analytical mode of hearing that both presupposed and reproduced new ideas about the physical nature of sound.⁸⁹ As these ideas made their way into the hardware and software of modern sound technologies, Ohm's unmusical ear thereby also prefigures the many kinds of analytical, machine-based, and "unmusical" modes of "hearing" that are currently all around us.

⁸⁹ Following Jonathan Sterne, the unmusical ear might be called an "audile technique": "a set of practices of listening that . . . encouraged the coding and rationalization of what was heard." See Jonathan Sterne, *The Audible Past: Cultural Origins of Sound Reproduction* (Durham, N.C.: Duke Univ. Press, 2003), p. 23.